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APPLICATIONS OF I-FUNCTIONS OF SEVERAL VARIABLES IN STATISTCAL DISTRIBUTIONS

P. C. SREENIVAS[1], T. M. VASUDEVAN NAMBISAN[2] AND MAYA P. V.[3]
[1] Department of mathematics,
Payyanur College, Payyanur,
Kannur District, Kerala-670327, India
[2] Retired Professor & Head, Department of Mathematics,
NAS College, Kanhangad, India
[3] Department of mathematics,
Mahatma Gandhi College, Iritty, Kannur District, Kerala, India

Abstract

The aim of this paper is to derive generalized multivariate statistical distributions involving the density function as the I-functions. Special cases include the results given by Mohammed [4,p.164].

1. Introduction

Notations and Results used :

$(a)_n$ stands for $a(a+1)\cdots(a+n-1)$

$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)},\ n \geq 1$

${}_1(a_j; \alpha_j, A_j)_p$ stands for $(a_1; \alpha_1, A_1), (a_2; \alpha_2, A_2), \cdots, (a_p; \alpha_p, A_p)$

$n = \frac{\Gamma(n+1)}{\Gamma n},\quad n \geq 1.$

Key Words : *Multivariable I-functions, The probability density function, The cumulative distribution function, Characteristic function.*

Prathima [7, p. 38].

I-function of r-variables is defined and represented as,

$$I[z_1,\cdots,z_r]=I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} z_1 & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P :_1 (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots,(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ z_r & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q :_1 (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\theta_1(s_1)\cdots\theta_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}ds_1\cdots ds_r, \tag{1.1}$$

where $\phi(s_1,\cdots,s_r)$ and $\theta_i(s_i), i=1,2,\cdots,r$ are given by,

$$\phi(s_1,\cdots,s_r)=\frac{\prod\limits_{j=1}^{N}\Gamma^{A_j}\left(1-a_j+\sum\limits_{i=1}^{r}\alpha_j^{(i)}s_i\right)}{\prod\limits_{j=1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum\limits_{i=1}^{r}\beta_j^{(i)}s_i\right)\prod\limits_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum\limits_{i=1}^{r}\alpha_j^{(i)}s_i\right)}, \tag{1.2}$$

$$\theta_i(s_i)=\frac{\prod\limits_{j=1}^{m_i}\Gamma_j^i(d_j^{(i)}-\delta_j^{(i)}s_i)\prod\limits_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod\limits_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)}, \tag{1.3}$$

The I-function of r-variables is analytic if

$$\Psi_i=\sum_{j-1}^{P}A_j\alpha_j^{(i)}-\sum_{j-1}^{Q}B_j\beta_j^{(i)}+\sum_{j-1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j-1}^{q_i}D_j^{(i)}\delta_j^{(i)}\le 0, i=1,2,\cdots,r.$$

The integral (1.1) converges absolutely if $|arg(z_i)|<\frac{1}{2}\Delta_i\pi,\quad i=1,2,\cdots,r$, where

$$\begin{aligned}\Delta_i \;=\; & \Bigg(-\sum_{j-n+1}^{P}A_j\alpha_j^{(i)}-\sum_{j-1}^{Q}B_j\beta_j^{(i)}+\sum_{j-1}^{m_i}D_j^{(i)}\delta_j^{(i)}-\sum_{j-m_i+1}^{q_i}D_j^{(i)}\delta_j^{(i)} \\ & +\sum_{j=1}^{n_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\gamma_j^{(i)}\gamma_j^{(i)}\Bigg)>0\end{aligned} \quad . \tag{1.4}$$

On taking $D_j^{(i)}=1$ $(j=1,2,\cdots,m_i,i=1,2,\cdots,r)$ in (1.1), then I-function will be denoted by

$$\overline{I}[z_1, \cdots, z_r] = I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right] \tag{1.5}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \overline{\theta}_1(s_1) \cdots \overline{\theta}_r(s_r) \phi(s_1, \cdots, s_r) Z_1^{s_1} \cdots Z_r^{s_r} ds_1 \cdots ds_r,$$

where

$$I_1 = {}_1(a_j; al_j^{(1)}, \cdots,]al_j^{(r)}; A_j)_P :_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots ; {}_1(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{p_r}$$

$$I_2 = {}_1(b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}; B_j)_Q : {}_1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};$$

$${}_{m_i+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \cdots ; {}_1(d_j^r, \delta_j^r; 1)_{m_r} :_{m_i+1} (d_j^{(r)}, \delta_j^{(r)}; nD_j^{(r)})_{q_r}$$

$$\overline{\theta}_i(s_r) = \frac{\prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - \delta_j^{(i)} s_i) \prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}(1 - c_j^{(i)} + \gamma_j^{(i)} s_i)}{\prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}(1 - d_j^{(i)} + \delta_j^{(i)} s_i) \prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}(c_j^{(i)} - \gamma_j^{(i)} s_i)},$$

where $i = 1, 2, \cdots, r$.
The integral (1.5) converges absolutely if $|arg(z_i)| < \frac{1}{2}\Delta_i'\pi, i = 1, 2, \cdots, r$, where

$$\Delta_i' = \left(- \sum_{j-n+1}^{P} A_j \alpha_j^{(i)} - \sum_{j-1}^{Q} B_j \beta_j^{(i)} + \sum_{j-1}^{m_i} \delta_j^{(i)} - \sum_{j-m_i+1}^{q_i} D_j^{(i)} \delta_j^{(i)} + \sum_{j-1}^{n_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j-n_i+1}^{p_i} C_j^{(i)} \gamma_j^{(i)} \right) > 0, i = 1, 2, \cdots, r. \tag{1.6}$$

When $N = P = Q = 0$, in $\overline{I}$ function of r-variables $\overline{I}[z_1, \cdots, z_r]$ breaks in to the product of r functions. The probability density function $f(x)$ of random variable X is defined as

$$\int_{-\infty}^{+\infty} f(x)dx = 1, \quad f(x) \geq 0, \quad \forall\ x. \tag{1.7}$$

The cumulative distributioin function of X is given by

$$F(x) = \int_{-\infty}^{+\infty} f(t)dt \ \ provided \ \ F(-\infty) = 0, F(\infty) = 1 \ \& \ \ P(a \leq X \leq b) = F(b) - F(a). \tag{1.8}$$

The characterestic function of X with density function

$$f(x) \ \ is, \ \ \phi(t) = E(e^{i\alpha}) = \int_{-\infty}^{+\infty} e^{i\alpha} f(x)dx, \ \ provided \ \ \phi(0) = 1, |\phi(t)| \leq 1, \tag{1.9}$$

$\phi(t)$ is continuous in t and is defined in every finite interval.

2. Main Results

The probability density function $f(x)$ of a family of finite distributiion is given by

$$\begin{aligned} f(x) &= \left(\frac{1}{K}\right) x^{d-1^-} I^{m_1 n_1}_{p,q_1}\left[z_1 x^{h_1} \left| \begin{array}{l} {}_1(c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1} \\ \\ {}_1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ {}_{m_i+1}(d_j^{(1)}\delta_j^{(1)}; D_j^{(1)})_{q_1} \end{array} \right. \right] \\ & \quad \cdots \overline{I}^{m_r,n_r}_{p_r,q_r}\left[z_r x^{h_r} \left| \begin{array}{l} {}_1(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{p_r} \\ \\ {}_1(d_j, \delta_j^{(r)}; 1)_{m_r};\ {}_{m_r+1}(d_j^{(r)}\delta_j^{(r)}; D_j^{(r)})_{q_1} \end{array} \right. \right], 0 < x < 1 \\ &= 0, \quad elsewhere \end{aligned} \tag{2.1}$$

The constant K is given by

$$K = \overline{I}^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array} \right], \tag{2.2}$$

where

$$\begin{aligned} I_1 &= (1-d, h_1, \cdots, h_r; 1)_1; A_j)_P:\ {}_1(c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots;\ {}_1(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{p_r} \\ I_2 &= (-d, h_1, \cdots, h_r; 1);\ {}_1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ {}_{m_1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1} \\ & \quad ; \cdots;\ {}_1(d_j^r, \delta_j^r; 1)_{m_r};\ {}_{m_r+1}(d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r} \end{aligned}$$

provided

1. (i) $d_k > 0, h_k > 0, k = 1, 2, \cdots, r.$
2. (ii) $Re(d, h_k d_j^{(k)}/\delta_j^{(k)}) > 0, \quad j = 1, 2, \cdots, m_k, k = 1, 2, \cdots, r$
3. (iii) $|arg(z_1)| < \frac{1}{2}\Delta_i'\pi, i = 1, 2, \cdots, r$ where

$$\begin{aligned} \Delta_i' &= \left(-\sum_{j=n+1}^{P} A_j \alpha_j^{(i)} - \sum_{j=1}^{Q} B_j \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{a_i} D_j^{(i)} \delta_j^{(i)} \right. \\ & \qquad \left. + \sum_{j-1}^{n_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j-n_i+1}^{p_i} C_j^{(i)} \gamma_j^{(i)} \right) > 0, i = 1, 2, \cdots, r. \end{aligned}$$

Proof : Consider the integral

$$\int_0^1 x^{d-1}\overline{I}[z_1x^{h_1},\cdots,z_rx^{h_r}]dx$$
$$=\int_0^1 x^{d-1}\left(\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)(z_1x^{h_1})^{s_1}\cdots(z_rx^{h_r})^{s_r}ds_1\cdots ds_r\right)dx$$
$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}\left(\int_0^1 x^{h_1s_1+\cdots+h_rs_r+d-1}dx\right)ds_1\cdots ds_r$$
$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\left(\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}\frac{1}{(h_1s_1+\cdots+h_rs_r+d)}\right)ds_1\cdots ds_r$$
$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\left(\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}\frac{\Gamma(h_1s_1+\cdots+h_rs_r+d)}{\Gamma(h_1s_1+\cdots+h_rs_r+d+1)}\right)$$
$$=I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+1:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2\end{array}\right].$$

where

$$I_1 = (1-d,h_1,\cdots,h_r;1)_1(a_j,\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r}$$
$$I_2 = (-d,h_1,\cdots,h_r1)\ {}_1(b_j;\beta_j^{(1)};\cdots,\beta_j^{(r)};B_j)_Q :_1 (d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};$$
$$;\cdots;\ {}_1(d_j^r,\delta_j^r;1)_{m_r};\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}.$$

Put $N=P=Q=0$. Then

$$\int_0^1 x^{d-1}(\overline{I}[z_1x^{h_1}]\overline{I}[z_2x^{h_2}]\cdots\overline{I}[z_rx^{h_r}])dx=\overline{I}^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots;p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2\end{array}\right]$$

$$I_1 = (1-d,h_1,\cdots,h_r;1)_1\ (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};$$
$$\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r}$$
$$I_2 = (-d,h_1,\cdots,h_r;1)_1\ (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q:\ {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m+1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};$$
$$\cdots;\ {}_1(d_j^r,\delta_j^r;1)_{m_r};\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}.$$

3. Properties

(i) The cumulative distribution function $P(x)$w is given by

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(t)dt \\ &= \int_{0}^{x} f(t)dt \\ &= \left(\frac{1}{k}\right) x^d I^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots,p_r,q_r}\left[\begin{array}{c|c} z_1x^{h_1} & I_1 \\ \vdots & \\ z_rx^{h_r} & I_2 \end{array}\right], \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} I_1 &= (1-d, h_1, \cdots, h_r; 1)_1; (A_j)_P :_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots ;\ _1(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{p_r} \\ I_2 &= (-d, h_1, \cdots, h_r; 1);\ \ _1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ _{m+1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \\ &\quad \cdots ;\ _1(d_j^r, \delta_j^r; 1)_{m_r};\ _{m_r+1}(d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r}. \end{aligned}$$

Provided the conditions similar to that of (2.1) are satisfied.

The proof of (3.1) is similar to that of (2.1).

(ii) The characterestic function is given by

$$\begin{aligned} \phi(t) &= \int_{-\infty}^{+\infty} e^{itx} f(x)dx \\ &= \int_{0}^{1} e^{itx} f(x)dx \\ &= \left(\frac{1}{k}\right) \sum \frac{(it)^n}{n!} I^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots,p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right], \\ I_1 &= (1-n-d, h_1, \cdots, h_r; 1)_1;_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots ;\ _1(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{p_r} \\ I_2 &= (-n-d, h_1, \cdots, h_r; 1);\ \ _1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ _{m+1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \\ &\quad \cdots ;\ _1(d_j^r, \delta_j^r; 1)_{m_r};\ _{m_r+1}(d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{q_r}. \end{aligned}$$

Provided the conditions similar to that of (2.1) and $f(x)$ and K are defined by (2.1) and (2.2) respectively.

Proof : Consider the integral,

$$\int_0^1 e^{itx}\left(\frac{1}{K}\right)x^{d-1}(\overline{I}[z_1x^{h_1}z_2x^{h_2}\cdots z_rx^{h_r}])$$
$$=\frac{1}{K}\int_0^1 x^{d-1}\left(\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}^{-}\theta_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)(z_1x^{h_1})\cdots(z_rx^{h_r})ds_1\cdots ds_r\right)dx$$
$$=\left(\frac{1}{K}\right)\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)z_1^{s_1}\cdots z_r^{s_r}$$
$$\left(\int_0^1 x^{h_1s_1+\cdots+h_rs_r+d-1}dx\sum\frac{(itx)^n}{n!}\right)ds_1\cdots ds_r$$
$$\left(\frac{1}{K}\right)\sum\frac{(it)^n}{n!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}(\overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\phi(s_1,\cdots,s_r)$$
$$z_1^{s_1}\cdots z_r^{s_r}\left(\frac{\Gamma(n+h_1s_1+\cdots+h_rs_r+d)}{\Gamma(h_1s_1+\cdots+h_rs_r+d+1)}\right)ds_1\cdots ds_r$$
$$=I^{0,N+1:m_1,n_1;\cdots;m_r,n_r}_{P+1,Q+1:p_1,q_1;\cdots,p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2\end{array}\right],$$

where

$$\begin{aligned} I_1 &= (1-n-d,h_1,\cdots,h_r;1)_{1;1}\,(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ I_2 &= (-n-d,h_1,\cdots,h_r;1);\ \ {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m+1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1}; \\ &\quad \cdots;\ {}_1(d_j^r,\delta_j^r;1)_{m_r};\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}. \end{aligned}$$

4. Special cases

When $r=2$, the probability density function $f(x)$ of a family of finite distribution is given by,

$$\begin{aligned} f(x) &= \left(\frac{1}{K}\right)x^{d-1}\overline{I}^{m_1,n_1}_{p_1,q_1}\left[z_1x^{h_1}\left|\begin{array}{l} {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1} \\ \\ {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_j^{(1)}\delta_j^{(1)};D_j^{(1)})_{q_1}\end{array}\right.\right] \\ &\quad \overline{I}^{m_2,n_2}_{p_2,q_2}\left[z_2x^{h_2}\left|\begin{array}{l} {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_1} \\ \\ {}_1(d_j,\delta_j^{(r)};1)_{m_1};\ {}_{m_1+1}(d_j^{(r)}\delta_j^{(r)};D_j^{(r)})_{q_r}\end{array}\right.\right],0<x<1 \\ &= 0\ \ elsewhere. \end{aligned} \tag{4.1}$$

When $r = 2$, the cumulative distribution $F(x)$ is given by

$$F(x) = \frac{1}{k} x^d I^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots,p_r,q_r} \left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array} \right] \tag{4.2}$$

$$\begin{aligned} I_1 &= (1-d, h_1, h_2; 1)_1 A_j)_P;_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1}; \cdots ;\ _1(c_j^{(2)}, \gamma_j^{(2)}; C_j^{(2)})_{p_2} \\ I_2 &= (-d, h_1, h_2; 1);\ \ _1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ _{m+1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \\ &\quad \cdots ;\ _1(d_j^{(2)}, \delta_j^{(2)}; 1)_{m_2};\ _{m_2+1}(d_j^{(2)}, \delta_j^{(2)}; D_j^{(2)})_{q_2}. \end{aligned}$$

When $r = 2$, the characteristic function is given by

$$\phi(t) = \left(\frac{1}{k}\right) \sum \frac{(it)^n}{n!} I^{0,1:m_1,n_1;\cdots;m_r,n_r}_{1,1:p_1,q_1;\cdots,p_r,q_r} \left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array} \right] \tag{4.3}$$

$$\begin{aligned} I_1 &= (1-n-d, h_1, h_2; 1);_1 (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{p_1};\ _1(c_j^{(2)}, \gamma_j^{(2)}; C_j^{(2)})_{p_2} \\ I_2 &= (-n-d, h_1, h_2; 1);\ \ _1(d_j^{(1)}, \delta_j^{(1)}; 1)_{m_1};\ _{m+1+1}(d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{q_1}; \\ &\quad _1(d_j^2, \delta_j^2; 1)_{m_2};\ _{m_2+1}(d_j^{(2)}, \delta_j^{(2)}; D_j^{(2)})_{q_2}. \end{aligned}$$

When $C_j^{(1)} \cdots C_j^{(r)} = 1; D_j^{(1)} \cdots D_j^{(r)} = 1$, (2.1) reduces to the result given by Mohammed [4, p. 164].

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